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EQUISTRESSED REINFORCEMENT OF KIRCHHOFF PLATES FOR ELASTOPLASTIC TRANSVERSE BENDING†

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The problem of equistressed reinforcement of Kirchhoff plates with fibres of constant cross-section in the case of elastoplastic transverse bending is formulated. A qualitative analysis of the system of resolvents is carried out. The possibility of the existence of several alternative solutions which can be controlled by a redistribution of the reinforcement densities is demonstrated. An analytic solution of the problem is obtained for the case of the cylindrical bending of a rectangular elongated plate. Calculations are carried out for boroaluminium, which show that the carrying capacity of equistressed reinforced plates with elastoplastic bending is several times greater than in the case of purely elastic bending. © 2004 Elsevier Ltd. All rights reserved.

The requirement that the fibres should be equistressed along their trajectories, which enables one to utilize the carrying capacity of high-strength equipment to the greatest extent and to construct reliable structures even when a low-strength binder is used, serves as one of the most natural strength criteria in the rational design of composite structures. Due to the current interest in the problem of equistressed reinforcement, many papers [1–5, etc.] are concerned with this. However, up to now, it has been assumed when investigating the problem of the equistressed reinforcement of bending plates that all of the phases of the composition behave in a linear elastic manner [4, 5], that is, no account has been taken of the actual behaviour of the phase materials beyond the yield point and no estimation has been made of the effectiveness of the carrying capacity of real fibres in the treatment of the equistressed reinforcement problem in an elastic formulation. In this connection, the aim of this paper is to give a mathematical formulation and qualitative analysis of the problem of the equistressed reinforcement of plates with elastoplastic transverse bending and, also, a comparison, based on specific examples, of the carrying capacity of plates with equistressed reinforcement in the case of purely elastic and elastoplastic bending.

1. THE INITIAL EQUATIONS OF THE PROBLEM OF THE EQUISTRESSED REINFORCEMENT OF PLATES WITH ELASTOPLASTIC TRANSVERSE BENDING

We shall consider the purely elastic and elastoplastic transverse bending of Kirchhoff plates of constant thickness H, consisting of an isotropic matrix and a thin filament, homeogeneous, high modulus reinforcement of constant cross-section which is embedded into it. It is assumed that the plate has a regular and quasi-homogeneous structure throughout its thickness, that the action of heat is ignored and that the deflections are small. All of the phases of the composition can behave in a linearly elastic or in an inelastic manner. The way in which a plate is loaded is assumed to be quasistatic and simple and the relations of the theory of elastoplastic deformations [6, 7] are therefore used to describe the non-linearly elastic or inelastic behaviour of the phase materials. The requirement that fibres of all of the groups in the whole of the domain G occupied by the plate in the plan view should be equistressed emerges as the rational design criterion.

The plate is considered in a rectangular Cartesian system of coordinates x_1x_2z; the plane x_1x_2 is made coincident with the middle plane of the plate before it is bent and the z-axis is perpendicular to the middle plane. The plate is reinforced with N groups of fibres (which are possibly of a different physical nature)) which are laid in planes parallel to the plane x_1x_2.

In order to formulate the problem of the equistressed reinforcement of transversely bent Kirchhoff plates, it is necessary to use the well-known equilibrium equations in the shearing forces F_i and torques M_{ij} [7]

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$$F_{1,1}+F_{2,2}+p = 0, \quad M_{i1,1}+M_{i2,2} = F_i - m_i, \quad i = 1,2 \tag{1.1}$$

the relation between the mean stresses in the composition σ_{ij} and the moments M_{ij}

$$M_{ij} = \int_{-H/2}^{H/2} \sigma_{ij} z dz, \quad i,j = 1,2 \tag{1.2}$$

the relations between the bending deformations ε_{ij} and the deflection w

$$\varepsilon_{ij} = -zw_{,ij}, \quad i,j = 1,2 \quad (|z| \le H/2) \tag{1.3}$$

and, also, the expressions for the average stresses σ_{ij} in terms of the stresses in the phase materials (a model of a layer reinforced with "one-dimensional' fibres [8] is used)

$$\begin{gathered} \sigma_{ij} = a\sigma_{mij} + \sum_k \sigma_k \omega_k l_{ki} l_{kj} \quad (i,j = 1,2) \\ l_{k1} = \cos\psi_k, \quad l_{k2} = \sin\psi_k, \quad a = 1-\sum_k \omega_k \end{gathered} \tag{1.4}$$

where p and m_i are the distributed transverse load and the external bending moments respectively, σ_{mij} and σ_k are the stresses in the binding matrix and in the kth group of the reinforcement respectively, ω_k and ψ_k are the intensity and the angle (measured from the x_1 direction) of the reinforcement with a fibre of the kth group, summation is carried out from 1 to N and the subscript i after a comma denotes partial differentiation with respect to the variable x_i, etc.

We shall assume that the tensile and compression diagrams of the phase materials are identical and have a linear reinforcement. The relation between the stress σ_k and the longitudinal deformation ε_k of the kth reinforcement group has the form [8]

$$\sigma_k = \begin{cases} E_k\varepsilon_k, & |\varepsilon_k| \le \varepsilon_{sk} = \sigma_{sk}/E_k \\ \operatorname{sign}(\varepsilon_k)\sigma_{sk} + E_{sk}(\varepsilon_k - \operatorname{sign}(\varepsilon_k)\varepsilon_{sk}), & \varepsilon_{sk} < |\varepsilon_k| \le \varepsilon_{*k} \equiv \varepsilon_{pk} \end{cases} \tag{1.5}$$

where σ_{sk} is the yield point of the material of the kth group of fibres, E_k and E_{sk} are the moduli of elasticity and of strain hardening of the material of the kth group of fibres, and ε_{sk} and ε_{*k} are the deformations corresponding to the yield point and the ultimate strength σ_{pk} of the material of the kth group of fibres, respectively.

The relation between the deformations of the plate ε_{ij} and the deformations of the fibres ε_k is determined, within the framework of a model one-dimensional fibres, by the relations [8]

$$\varepsilon_k = \varepsilon_{11}\cos^2\psi_k + \varepsilon_{22}\sin^2\psi_k + \varepsilon_{12}\sin 2\psi_k, \quad k = 1,2,\ldots,N \tag{1.6}$$

According to relations (1.3), (1.5) and (1.6), the maximum values of the stresses in the reinforcement with respect to their modulus are attained on the faces of the plate ($z = \pm H/2$) and therefore, to ensure that there is no ambiguity, we will specify the equistressed reinforcement condition solely on the upper side of the plate ($z = H/2$).

$$\sigma_k(x_1, x_2, H/2) = \sigma_{0k} = \text{const}, \quad k = 1,2,\ldots,N \tag{1.7}$$

where σ_{0k} is the stress value in the kth group of fibres on the upper side of the plate (on the lower side, $\sigma_k(x_1, x_2, -H/2) = -\sigma_{0k} = \text{const}$).

If $|\sigma_{0k}| \le \sigma_{sk}$. it follows from relations (1.3) and (1.5)–(1.7) that

$$\sigma_k(x_1, x_2, z) = 2z\sigma_{0k}/H \quad (|z| \le H/2) \tag{1.8}$$

If, however, $|\sigma_{0k}| > \sigma_{sk}$, it follows from the same relations that

$$\sigma_k(x_1, x_2, z) = \begin{cases} E_k z e_k = 2z\operatorname{sign}(\sigma_{0k})\sigma_{sk}/h_k, & |z| \le h_k/2 \\ \operatorname{sign}(z\sigma_{0k})\sigma_{sk} + E_{sk}[ze_k - \operatorname{sign}(z\sigma_{0k})\varepsilon_{sk}], & h_k/2 < |z| \le H/2 \end{cases} \tag{1.9}$$

where $h_k/2$ are the absolute magnitudes of the z coordinates of the boundaries between the elastic and inelastic layers in the kth group of reinforcement (that is, when $|z| \le h_k/2$, a fibre of the kth group behaves in an elastic manner and, when $h_k/2 < |z| \le H/2$, it behaves in an inelastic manner), and e_k is the bending parameter of the middle plane of the plate in the direction of the reinforcement with a fibre of the kth group which, according to relations (1.3) and (1.6), has the expression

$$e_k = -w_{,11}\cos^2\psi_k - w_{,22}\sin^2\psi_k - w_{,12}\sin 2\psi_k, \quad k = 1, 2, \dots, N \tag{1.10}$$

On the other hand, it follows from the equistressed reinforcement condition (1.7) and expressions (1.3), (1.5), (1.6) and (1.10) that

$$e_k = \begin{cases} 2\sigma_{0k}/(HE_k) = \text{const}, & |\sigma_{0k}| \le \sigma_{sk} \\ \dfrac{2}{HE_{sk}}[\sigma_{0k} - \operatorname{sign}(\sigma_{0k})(\sigma_{sk} - E_{sk}\varepsilon_{sk})] = \text{const}, & |\sigma_{0k}| > \sigma_{sk} \end{cases} \tag{1.11}$$

Hence, instead of the equistressed reinforcement condition (1.7), the geometrical condition

$$e_k = \text{const}, \quad k = 1, 2, \dots, N \tag{1.12}$$

can be used. The value of e_k is given by expressions (1.10) and (1.11).

By virtue of the linear distribution of the deformations throughout the thickness of the plate (1.3) and according to its meaning, the quantities h_k in formula (1.9) are given by the equalities

$$h_k = \begin{cases} H = \text{const}, & |\sigma_{0k}| \le \sigma_{sk} \\ 2\varepsilon_{sk}/|e_k| = \text{const}, & |\sigma_{0k}| > \sigma_{sk} \end{cases} \tag{1.13}$$

that is, by virtue of the equistressed reinforcement condition (1.11), $h_k = \text{const}$ everywhere in the domain G.

The linearly elastic behaviour of the binder material is determined by Hooke's law

$$\sigma_{mii} = Ea_1(\varepsilon_{ii} + \nu\varepsilon_{jj}), \quad \sigma_{mij} = Ea_2\varepsilon_{ij}, \quad j = 3 - i, \quad i = 1, 2 \quad (|z| \le h/2) \tag{1.14}$$

and the intensity of the strains ε_0 in this case is equal to [7]

$$\varepsilon_0 = \frac{2}{\sqrt{3}}\sqrt{a_3(\varepsilon_{11}^2 - a_4\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}^2) + \varepsilon_{12}^2} \tag{1.15}$$

where

$$a_1 = \frac{1}{1-\nu^2}, \quad a_2 = \frac{1}{1+\nu}, \quad a_3 = \frac{1-\nu+\nu^2}{3(1-\nu)^2}, \quad a_4 = \frac{1-4\nu+\nu^2}{1-\nu+\nu^2} \tag{1.16}$$

E and ν are the modulus of elasticity and Poisson's ratio of the binder.

The non-linearly elastic and inelastic behaviour of the binder material is defined by the fundamental relations of the theory of elastoplastic deformations [6, 7], which have been simplified by assuming that the material is incompressible (while not affecting the essence of the problem, there are significant difficulties associated with taking account of compressibility in the inelastic behaviour of the binder of a bending plate even in the simple case of the deformation of a material without strengthening [6, 7]). In the case when a strain diagram with linear strengthening is used, the relations between the stresses σ_{mij} and strains ε_{ij} beyond the limits of linear elasticity when there is no dilatation has the form

$$\begin{aligned} \sigma_{mii} &= \frac{2}{3\varepsilon}[\sigma_s + E_*(\varepsilon - \varepsilon_*)](2\varepsilon_{ii} + \varepsilon_{jj}) \\ \sigma_{mij} &= \frac{2}{3\varepsilon}[\sigma_s + E_*(\varepsilon - \varepsilon_*)]\varepsilon_{ij}; \quad j = 3 - i, \quad i = 1, 2 \end{aligned} \tag{1.17}$$

where σ_s and E_* are the yield point and the strengthening modulus of the binder material, which are known from the strain diagram [6], ε_* is the strain corresponding to σ_s in the strain diagram and, assuming the binder material is incompressible, the strain intensity ε has the expression [7]

$$\varepsilon^2 = 4(\varepsilon_{11}^2 + \varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}^2 + \varepsilon_{12}^2)/3 \tag{1.18}$$

We will now introduce into the treatment the positive quantities e_0 and e, which are composed of the parameters $w_{,ij}$ for the distortion of the median plane of the plate in the same way as for the intensities ε_0 and ε in (1.15) and (1.8) respectively

$$e_0 = \frac{2}{\sqrt{3}}\sqrt{a_3(w_{,11}^2 - a_4 w_{,11}w_{,22} + w_{,22}^2) + w_{,12}^2} \tag{1.19}$$

$$e^2 = 4(w_{,11}^2 + w_{,11}w_{,22} + w_{,22}^2 + w_{,12}^2)/3 \tag{1.20}$$

It then follows from relations (1.3), (1.15) and (1.18)–(1.20) that

$$\varepsilon_0 = |z|e_0(x_1, x_2), \quad \varepsilon = |z|e(x_1, x_2) \tag{1.21}$$

In the case of an elastoplastic stressed state in the outer layers of the binder adjacent to the forces, the binder material behaves in an inelastic manner while the middle layer still remains elastic. Consequently, if $h/2$ is the absolute value of the z coordinate of the boundaries between the elastic and inelastic in the binder, then the stress intensity

$$\sigma < \sigma_s (0 \le |z| < h/2), \quad \sigma \ge \sigma_s \quad (h/2 \le |z| \le H/2) \tag{1.22}$$

In the elastic layer of the binder, the stress intensity is equal to [6, 7]

$$\sigma = 3G_m\varepsilon_0 = 3G_m|z|e_0, \quad G_m = E/[2(1+\nu)] \tag{1.23}$$

where G_m is the shear modulus of the bonding matrix. At the boundaries $|z| = h/2$ between the elastic and inelastic layers in the binder $\sigma = \sigma$, and it therefore follows from (1.23) that

$$h_*/2 = \sigma_s/(3G_m e_0) \tag{1.24}$$

whence

$$h = \begin{cases} H = \text{const}, & h_* \ge H \\ h_*, & h_* < H \end{cases} \tag{1.25}$$

Relations (1.9), (1.24) and (1.25) determine the thickness h of the elastic layer in the binder in terms of the second derivatives of the deflection while, in the general case, $h \ne$ const.

Finally, when account is taken of expressions (1.3), (1.14), (1.17) and (1.21), the stresses in the binder σ_{mij} are given by the relations

$$\sigma_{mii} = -zEa_1(w_{,ii} + \nu w_{,jj}), \quad \sigma_{mij} = -zEa_2 w_{,ij} \quad (|z| \le h/2) \tag{1.26}$$

$$\begin{aligned} &\sigma_{mii} = -A(z)(2w_{,ii} + w_{,jj}), \quad \sigma_{mij} = -A(z)w_{,ij}, \quad j = 3-i, \quad i = 1,2 \\ &A(z) = 2[zE_* + \operatorname{sign}(z)(\sigma_s - E_*\varepsilon_*)/e]/3, \quad h/2 < |z| \le H/2 \end{aligned} \tag{1.27}$$

The quantities a_1 and a_2 are defined by formulae (1.16).

Substituting relations (1.4) into expression (1.2) and taking account of the representation (1.9), (1.13) and (1.25)–(1.27), in a plate with an equistressed reinforcement structure we obtain the following expressions for the moments in terms of the deflection of the plate

$$\begin{aligned} M_{ij} = {} & aM_{mij} + \sum_k \omega_k l_{ki} l_{kj} \times \\ & \times [E_k e_k h_k^3 + E_{sk} e_k (H^3 - h_k^3) + 3\operatorname{sign}(\sigma_{0k})(\sigma_{sk} - E_{sk}\varepsilon_{sk})(H^2 - h_k^2)]/12, \quad i,j = 1,2 \end{aligned} \tag{1.28}$$

$$\begin{aligned}
M_{\mathrm{m}ij} &= \int_{-H/2}^{H/2} \sigma_{\mathrm{m}ij} z dz, \quad i, j = 1, 2 \\
M_{\mathrm{m}ii} &= -[E a_1 h^3 (w_{,ii} + \nu w_{,jj}) + B(2w_{,ii} + w_{,jj})]/12 \\
M_{\mathrm{m}ij} &= -(E a_2 h^3 + B) w_{,ij}/12, \quad j = 3 - i, \quad i = 1, 2 \\
B &= 2[(H^3 - h^3) E_*/3 + (\sigma_s - E_* \varepsilon_*)(H^2 - h^2)/e]
\end{aligned} \tag{1.29}$$

If $|\sigma_{0k}| \le \sigma_{sk}$, $h_* \ge H$ (see (1.13), (1.24) and (1.25)), it is necessary to put $h_k = h = H$ in relations (1.28) and (1.29), after which we obtain expressions for the moments in the case of purely elastic bending of a plate with equistressed reinforcement.

The condition for the cross-sections of the fibres to be constant [3]

$$(\omega_k \cos\psi_k)_{,1} + (\omega_k \sin\psi_k)_{,2} = 0, \quad k = 1, 2, \ldots, N \tag{1.30}$$

has to be added to the equilibrium equations (1.1), the expressions for the moments (1.28) and (1.29) the equistressed reinforcement conditions (1.10)–(1.12) and Eq. (1.24) which, when relations (1.19) and (1.25) are taken into account, determines the thickness of the elastic layer in the binder.

Suppose the domain G is bounded by a contour Γ. Then, static boundary conditions with respect to the bending moment [7] can be specified on one side of this contour (which we shall denote by Γ_p)

$$\begin{aligned}
M_{11} n_1^2 + M_{22} n_2^2 + 2M_{12} n_1 n_2 &= M_n \\
n_1 = \cos\beta, \quad n_2 = \sin\beta, &\quad (x_1, x_2) \in \Gamma_p
\end{aligned} \tag{1.31}$$

and with respect to the reduced transverse Kirchhoff force

$$\begin{aligned}
F_1 n_1 + F_2 n_2 + \partial_\tau(M_{n\tau}) &= F_{nz}, \quad (x_1, x_2) \in \Gamma_p \\
M_{n\tau} &= (M_{22} - M_{11}) n_1 n_2 + M_{12}(n_1^2 - n_2^2) \\
\partial_\tau(M_{n\tau}) &= -n_2 M_{n\tau,1} + n_1 M_{n\tau,2}
\end{aligned} \tag{1.32}$$

while, on the other side (which we shall denote by Γ_u), the kinematic boundary conditions

$$w(\Gamma_u) = w_0, \quad w_{,1} n_1 + w_{,2} n_2 = \theta_n, \quad (x_1, x_2) \in \Gamma_u \tag{1.33}$$

can be specified, where M_n and F_{nz} are the bending moment and the reduced transverse Kirchhoff force specified on Γ_p, w_0 and θ_n are the deflection in Γ_u and the derivative of the deflection with respect to the direction of the outward normal to the contour which is given by the angle β, and ∂_τ is a derivative along the contour. (Boundary conditions which are mixtures of (1.31)–(1.33) such as the condition of free support, for example, can also be specified on the contour Γ.)

On the part of the contour Γ (which we shall denote by Γ_k) at which the fibres belonging to the kth group enter into the domain G, it is necessary to specify boundary conditions for the reinforcement intensities [4]

$$\omega_k(\Gamma_k) = \omega_{0k}, \quad k = 1, 2, \ldots, N \tag{1.34}$$

where ω_{0k} are functions, defined in Γ_k.

The solution of the problem of the equistressed reinforcement of bending plates must satisfy the physical constraints [1–3]

$$0 \le \omega_k, \quad k = 1, 2, \ldots, N, \quad \sum_k \omega_k \le \omega_* \le 1 \tag{1.35}$$

and the strength constraints [6–8]

$$\begin{aligned}
\sigma(x_1, x_2, \pm H/2) \le \sigma_{\mathrm{m}}, &\quad |\sigma_{0k}| \le \sigma_k^* = \min(\sigma_k^-, \sigma_k^+) \\
\sigma_{\mathrm{m}} > 0, \quad \sigma_k^\pm > 0, &\quad k = 1, 2, \ldots, N
\end{aligned} \tag{1.36}$$

where $\omega_* = \text{const}$ is the permissible overall reinforcement intensity, σ_m is the breaking point of the bonding matrix, which is equal to the yield point σ_s in the case of purely elastic bending or to the ultimate strength $\sigma_* \equiv \sigma_p$ in the case of elastoplastic bending, σ_k^- and σ_k^+ are the breaking points of the kth group of fibres under compression and tension respectively (under the action of compressive loads, a certain form of instability of the fibres can occur and, hence, in the general case, $\sigma_k^- \neq \sigma_k^+$).

The generally known static and kinematic conditions for the matching of the solution [6, 7] hold at the boundaries Γ_e^p between the purely elastic G_e and elastoplastic G_p zones in the binder, that is, when $h_* = H$ (see (1.24)). By virtue of the linear distribution of the stresses $\sigma_{\mathrm{m}ij}$ throughout the thickness of the plate in the purely elastic zone in the binder, it follows from the first strength constraint (1.36), when $\sigma_\mathrm{m} = \sigma_s$, that the above-mentioned boundary is determined by the equation [7]

$$\Gamma_e^p\colon M_{\mathrm{m}11}^2 - M_{\mathrm{m}11}M_{\mathrm{m}22} + M_{\mathrm{m}22}^2 + 3M_{\mathrm{m}12}^2 = H^4\sigma_s^2/36 = \text{const} \tag{1.37}$$

Hence, when formulating the problem of the equistressed reinforcement of bending plates with fibres of constant cross-section, it is necessary to use equations and relations (1.1), (1.10)–(1.12), (1.24), (1.25) and (1.28)–(1.30). In the contour Γ, which bounds the domain G, the static boundary conditions (1.31) and (1.32), the kinematic boundary conditions (1.33) or mixed boundary conditions from (1.31)–(1.33) and the edge conditions (1.34) can be specified. The boundary between the purely elastic and the elastoplastic zones in the binder, when there is a continuous change of the equistressed reinforcement structure in it, is defined by Eq. (1.37). The solution of the equistressed reinforcement problem must satisfy the physical constraint (1.35) and strength constraint (1.36) and, moreover, by virtue of the Kirchhoff hypothesis and the assumption that the tensile and compression diagrams of the phase materials are identical, it is sufficient to satisfy inequalities (1.36) on the faces of the plate ($z = \pm H/2$).

2. THE SYSTEM OF RESOLVENTS FOR THE PROBLEM OF THE EQUISTRESSED REINFORCEMENT OF PLATES IN THE CASE OF ELASTOPLASTIC BENDING AND ITS QUALITATIVE ANALYSIS

In order to obtain the equilibrium equations for the equistressed reinforcement of a plate for bending, it is necessary to substitute relations (1.28) and (1.29) into Eqs (1.1) and to eliminate the transverse forces F_i from consideration. As a result, when account is taken of the condition for the cross-sections of the fibres to be constant (1.30), we will have the following equilibrium equation

$$\begin{aligned} &H^2\sum_k \sigma_{*k}[\partial_n(\psi_k, A_k) - A_k^2/\omega_k] - C(w, \boldsymbol{\omega}, h) = \\ &= -12(p + m_{1,1} + m_{2,2}) \quad (\boldsymbol{\omega} = \{\omega_1, \omega_2, \ldots, \omega_N\}) \end{aligned} \tag{2.1}$$

where

$$A_k = \omega_k\partial_s(\psi_k, \psi_k), \quad k = 1, 2, \ldots, N \tag{2.2}$$

$$\partial_s(\gamma, \cdot) = \cos\gamma\frac{\partial(\cdot)}{\partial x_1} + \sin\gamma\frac{\partial(\cdot)}{\partial x_2}, \quad \partial_n(\gamma, \cdot) = -\sin\gamma\frac{\partial(\cdot)}{\partial x_1} + \cos\gamma\frac{\partial(\cdot)}{\partial x_2} \tag{2.3}$$

A_k are functions, which have been introduced for convenience and have the meaning of the curvature of the trajectories of the kth group of equistressed reinforcement multiplied by the intensity of the reinforcement of this group, and γ is a certain angle; in the zones where the binder material exhibits elastoplastic behaviour the differential operator C has the form

$$\begin{aligned} C(w, \boldsymbol{\omega}, h) = &\sum_{i=1,2} \{a[Ea_1(w_{,ii} + \nu w_{,jj})h^3 + B(2w_{,ii} + w_{,jj})]\}_{,ii} + \\ &+ 2[a(Ea_2h^3 + B)w_{,12}]_{,12}, \quad j = 3 - i \end{aligned} \tag{2.4}$$

in the zones of purely elastic behaviour of the binder material the operator C is simplified and is obtained from (2.4) when $h = H$, a_1, a_2, e and h are defined by expressions (1.16), (1.20), (1.24) and (1.25) respectively, and σ_{*k} is a constant quantity, defined by the expression

$$\sigma_{*k} = E_k e_{*k} h_{*k}^3 + E_{sk} e_{*k}(1 - h_{*k}^3) + 3\,\mathrm{sign}(\sigma_{0k})(E_k - E_{sk})\varepsilon_{sk}(1 - h_{*k}^2) = \mathrm{const} \tag{2.5}$$

The quantities h_{*k} and e_{*k} in (2.4) and (2.5) are defined by the following relations (see (1.11) and (1.13))

$$e_k = \frac{e_{*k}}{H} \tag{2.6}$$

$$e_{*k} = \begin{cases} 2\sigma_{0k}/E_k = \mathrm{const}, & |\sigma_{0k}| \le \sigma_{sk} \\ 2[\sigma_{0k} - \mathrm{sign}(\sigma_{0k})(\sigma_{sk} - E_{sk}\varepsilon_{sk})]/E_{sk} = \mathrm{const}, & |\sigma_{0k}| > \sigma_{sk} \end{cases}$$

$$h_k = h_{*k}H, \quad h_{*k} = \begin{cases} 1, & |\sigma_{0k}| \le \sigma_{sk} \\ 2\varepsilon_{sk}/|e_{*k}| = \mathrm{const} < 1, & (|\sigma_{0k}| > \sigma_{sk}) = E_k\varepsilon_{sk} \end{cases} \tag{2.7}$$

It follows from (2.5)–(2.7) that, in the case of the purely elastic deformation of the kth group of reinforcement ($|\sigma_{0k}| \le \sigma_{sk}$), the quantity σ_{*k} in (2.1) is double the stress in this reinforcement on the upper side of the plate ($z = H/2$).

The equistressed reinforcement conditions for the kth group of fibres (1.10) and (1.11), which we shall write, taking account of expressions (2.2) and (2.3), as

$$\partial_s(\psi_k, \partial_s(\psi_k, w)) - A_k\partial_n(\psi_k, w)/\omega_k = -e_{*k}/H = \mathrm{const} \tag{2.8}$$

have to be added to Eqs (2.1), (2.2) and (1.24). The condition for the cross-section of the fibres to be constant (1.30), which, when account is taken of the notation in (2.3), takes the form

$$\partial_s(\psi_k, \omega_k) + \omega_k\partial_n(\psi_k, \psi_k) = 0, \quad k = 1, 2, \ldots, N \tag{2.9}$$

has to be used finally to close system (2.1), (2.2), (1.24), (2.8).

In other to write the static boundary conditions (1.31) and (1.32) for the bendings, it is necessary to eliminate the force factors F_i and M_i from them by means of relations (1.1), (1.28) and (1.29). As a result, when account is taken of the condition for the cross-section of the fibres to be constant (1.3), we obtain the static boundary conditions on the edge Γ_p with respect to the moment

$$H^2\sum_k \sigma_{*k}\omega_k\cos^2(\psi_k - \beta) - D_M(w, \omega, h) = 12M_n, \quad (x_1, x_2) \in \Gamma_p \tag{2.10}$$

and with respect to the reduced Kirchhoff force

$$\begin{aligned} &-H^2\sum_k \sigma_{*k}A_k\sin(\psi_k - \beta) + H^2\sum_k \sigma_{*k}\partial_\tau[\omega_k\sin 2(\psi_k - \beta)]/2 - \\ &- C_F(w, \omega, h) - D_F(w, \omega, h) = 12(F_{nz} - m_1n_1 - m_2n_2), \quad (x_1, x_2) \in \Gamma_p \end{aligned} \tag{2.11}$$

where, in the subdomains G_p with elastoplastic behaviour of the binder material, the differential operators D_M, C_F and D_F are given by the expressions

$$\begin{aligned} D_M &= \sum_{i=1,2} a[W_i + B(2w_{,ii} + w_{,jj})]n_i^2 + 2aw_{,12}(Ea_2h^3 + B)n_1n_2 \\ C_F &= \sum_{i=1,2} \{a[W_i + B(2w_{,ii} + w_{,jj})]\}_{,i}n_i + \sum_{i=1,2} [a(Ea_2h^3 + B)w_{,ij}]_{,i}n_j \\ D_F &= \partial_\tau\left\{n_1n_2a\sum_{i=1,2}(-1)^i[W_i + B(2w_{,ii} + w_{,jj})] + \right. \\ &\left. + a(n_1^2 - n_2^2)(Ea_2h^3 + B)w_{,12}\right\}, \quad W_i = Ea_1(w_{,ii} + \nu w_{,jj})h^3; \quad j = 3 - i \end{aligned} \tag{2.12}$$

and in the subdomains G_e with purely elastic behaviour of the binder material, the differential operators D_M, C_F and D_F are obtained from expressions (2.12) when $h = H$, where ∂_τ is the operator of differentiation along the contour Γ_p, and

$$\partial_\tau(\cdot) = \partial_n(\beta, \cdot) \tag{2.13}$$

The kinematic boundary conditions (1.33), taking into account the notation used in ((2.3), take the form

$$w(\Gamma_u) = w_0, \quad \partial_s(\beta, w) = \theta_n \quad (x_1, x_2) \in \Gamma_u \tag{2.14}$$

Boundary conditions (1.34) remain unchanged.

The boundary Γ_e^p between the purely elastic and the elastoplastic zones in the binder is defined by the equation in deflections

$$H^2E^2\left\{a_1^2 \sum_{i=1,2} (w_{,ii} + \nu w_{,jj})^2 - a_1^2 \prod_{i=1,2} (w_{,ii} + \nu w_{,jj}) + 3a_2^2 w_{,12}^2\right\} = 4\sigma_s^2, \quad j = 3 - i \tag{2.15}$$

which is obtained from (1.37) taking expressions (1.29) into account.

Hence, in the subdomains G_p with elastoplastic behaviour of the binder material, the system of equations of the problem of the equistressed reinforcement of bending plates consists of $3N + 2$ equations (2.1), (2.2), (1.24), (2.8), and (2.9) and is closed with respect to the following unknown functions: the deflection w, the equistressed reinforcement parameters A_k, ψ_k and ω_k and the thickness of the elastic layer in the binder $h = h_*$. In the subdomains G_e with purely elastic behaviour of the binder material ($h = H$), the system of resolvents consists of $3N + 1$ equations (2.1), (2.2), (2.8) and (2.9) and is closed with respect to the unknown functions w, A_k, ψ_k and ω_k ($k = 1, 2, \dots, N$). The boundary and edge conditions (2.10), (2.11), (2.14) and (1.34) must be specified for unique integration of the system of resolvents on the edges. The boundary between the zones of purely elastic and elastoplastic behaviour of the binder materials when there is a continuous change in the equistressed reinforcement structure in its is defined by Eq. (2.15).

The system of equations of the equistressed reinforcement problem (2.1), (2.2), (1.24), (2.8), (2.9) which has been obtained and the static boundary conditions (2.10) and (2.11) corresponding to it show that the subproblems of the determining the deflection and the equistressed reinforcement parameters are related and they have to be solved simultaneously and that, as a whole, the system of equations and static boundary conditions is extremely non-linear. This non-linearity has a twofold origin: first, "structural" non-linearity (since the equistressed reinforcement parameters A_k, ψ_k and ω_k, which determine the structure of the material, are unknown functions) and, second, physical non-linearity in the zones where the binder material exhibits elastoplastic behaviour. All of this considerably complicates the qualitative analysis of the boundary-value problem of the equistressed reinforcement of bending plates and the development of methods for solving it.

The characteristic equation of the resolving system has the form

$$P(x_2') \prod_{k=1}^{N} (\sin\psi_k - x_2' \cos\psi_k) = 0 \tag{2.16}$$

The derivative $x_2' = dx_2/dx_1$ specifies the direction of the characteristic, and $P(x_2')$ is a fourth-order polynomial in x_2', the coefficients of which depend on the values of the unknown functions $w_{,ij}$, ω_k, ψ_k and h. In subdomains with purely elastic behaviour of the binder material, in which it is necessary to put $h = H$ in Eq. (2.4), the polynomial $P(x_2')$ is defined by the expression

$$P(x_2') = HaEa_1(1 + x_2'^2)^2 \prod_{k=1}^{N} \gamma_k +$$

$$+ \sum_k (\sigma_{m11} x_2'^2 - 2\sigma_{m12} x_2' + \sigma_{m22} + \varsigma_k^2 \sigma_{*k}) \omega_k \eta_k \varsigma_k \prod_{l=1}^{N} \gamma_l, \quad l \neq k$$

where

$$\varsigma_k = \sin\psi_k - x_2'\cos\psi_k, \quad \eta_k = \cos\psi_k + x_2'\sin\psi_k$$

$$\gamma_k = (w_{,22} - w_{,11})\sin 2\psi_k + 2w_{,12}\cos 2\psi_k; \quad k = 1, 2, \ldots, N$$

and σ_{mij} are defined by expressions (1.26) on specifying that $z = H/2$. In the subdomains with elastoplastic behaviour of the binder material, the expression for the polynomial $P(x_2')$ becomes very cumbersome due to the complex expression for the operator C in (2.4) when $h \neq H$ and we shall therefore not derive it here.

The factors in (2.16) under the product sign, indicate that the system of resolvents has N real characteristics which coincide with the trajectories of the equistressed reinforcement. Depending on the values of the unknown functions $w_{,ij}$, ω_k, ψ_k and h (that is, depending on the coefficients), the polynomial $P(x_2')$ can have a different number of real roots at different points of the domain G. Consequently, the system of resolvents of the problem of the equistressed reinforcement of bending plates is a quasi-linear system of the hybrid-composite type [9].

The solution of the problem of the equistressed reinforcement of plates in the case of elastoplastic bending possesses the same properties and special features as in the case of purely elastic bending [4]. On making the system of resolvents and boundary conditions dimensionless, by analogy with the procedure previously used in [5], a small parameter $\lambda = E/E_1$ can be separated out in the case of the dimensionless operators C, C_F, D_M and D_F in relations (2.1), (2.10), and (2.11). After this, the methods of perturbation theory, similar to those used earlier in [5], can be employed to solve the equistressed reinforcement problem.

We will now consider the question of the non-uniqueness of the solution of the problem of equistressed reinforcement of bending plates. On the one hand, the equistressed reinforcement problem possesses arbitrariness associated with the boundary conditions for the reinforcement intensities (1.34) and, the greater the numbers of groups of fibres which are used, the greater the number of these arbitrarinesses. By varying the functions ω_{0k} in the boundary conditions (1.34), it is possible to obtain bundles of solutions of the equistressed reinforcement problem from which schemes with the most acceptable mechanical, weight or technological properties can be chosen. On the other hand, by virtue of the substantially non-linear static boundary conditions (2.10) and (2.11) and the conditions for the equistressed character of the reinforcement (2.8) with respect to the functions ψ_k, the equistressed reinforcement problem can have several solutions even in the case of fixed functions ω_{0k} in the boundary conditions (1.34) which further extends the spectrum of solutions of this problem. (The possibility that several alternative solution of the equistressed reinforcement problem with fixed input data can exist is explained by the fact that this problem belongs to a series of inverse problems in the mechanics of a deformable solid [10].)

3. EQUISTRESSED REINFORCEMENT OF PLATES WITH CYLINDRICAL BENDING

We will investigate the case of the cylindrical bending of an elongated rectangular plate which enables us to obtain a solution of the equistressed reinforcement problem in analytic form. For this purpose, we consider an elongated rectangular plate (ideally, of infinite length) of width D orientated along the Ox_2 axis. Assuming that the load, the fixing and the reinforcement of the plate do not change in the longitudinal direction and neglecting local end effects, we find that the solution of the equistressed reinforcement problem will depend solely on the variable x_1. In this case, Eqs (2.1) and (2.9) can be integrated, after which the system of resolvents of the equistressed reinforcement problem takes the form

$$12M_{11}(x_1) \equiv H^2\sum_k \sigma_{*k}\omega_k\cos^2\psi_k - \left(1 - \sum_k \omega_k\right)[b_1(h)w'' + b_2(h)\operatorname{sign}(w'')] = 12P(x_1) \tag{3.1}$$

$$\omega_k\cos\psi_k = \omega_{*k} = \text{const} \neq 0, \quad k = 1, 2, \ldots, N \tag{3.2}$$

$$w''\cos^2\psi_k = -e_k/H = \text{const} \neq 0, \quad k = 1, 2, \ldots, N \tag{3.3}$$

$$h(x_1) = \begin{cases} H = \text{const}, & h_* \geq H \\ h_*, & h_* < H \end{cases}, \quad h_* = \frac{2\sigma_s}{Ea_1\sqrt{1 - \nu + \nu^2}|w''|} \tag{3.4}$$

where

(3.5)

P_0 and P_1 are integration constants which are determined from the static boundary conditions, ω_{*k} are integration constant which, apart from H, have the meaning of the overall area of the cross-sections of the kth group of fibres which intersect an area of unit length (along x_2) orthogonal to the x_1 direction [11] (ω_{*k} can be given instead of $\omega_{0k}(x_2)$ = const in boundary conditions (1.34)), and a prime denotes differentiation with respect to x_1.

It follows from system (3.1)–(3.4) that, in solving the equistressed reinforcement problem in the case of cylindrical bending, it is quite sufficient to embed two groups of reinforcement ($N = 2$) in the plate which are made of the same material ($E_1 = E_2, \sigma_{*1} = \sigma_{*2}, E_{s1} = E_{s2}, \varepsilon_{s1} = \varepsilon_{s2}, e_{*1} = e_{*2}$ and are packed with the same intensity $\omega_1 = \omega_2$, symmetrically in the direction of $x_1(\psi_1 = -\psi_2)$, Then, by means of (3.2) and (3.3), the functions w'' and ω_1 can be eliminated from Eqs (3.1) and (3.4), after which we obtain

(3.6)

(3.7)

In the case of purely elastic behaviour of the binder material ($h = H$), Eq. (3.6) is reduced to the form

(3.8)

When multiplied by $\cos^3\psi_1$, Eq. (3.8) reduces to a fourth-order algebraic equation in $\cos\psi_1$. Consequently, with the corresponding input data, this problem can have up to four different solutions which depend parametrically on ω_{*1}, that is, on the amount of reinforcement embedded in the plate.

In the case of elastoplastic behaviour of the binder material ($h = h_* < H$), Eq. (3.6), after multiplication by $\cos^3\psi_1$ and taking account of relations (3.7), reduces to a seventh-order algebraic equation in $\cos\psi_1$. This means that Eq. (3.6) can have up to seven different solutions which depend parametrically on ω_{*1}.

Of the whole set of real solutions of Eqs (3.6) and (3.8), only those which satisfy the physical constraints $\omega_1 > 0$, $\omega_* - 2\omega_1 \ge 0$, $|\cos\psi_1| \le 1$ and the strength constraints (1.36) will be solutions of the equistressed reinforcement problem.

We shall assume that the plate is rigidly clamped along the edge $x_1 = D$ and the static boundary conditions (2.10) and (2.11) when $\beta = \pi$ are specified on the edge $x_1 = 0$. If there are no external transverse loads ($p(x_1) = 0, F_{nz} = 0$), the case of purely cylindrical bending ($P(x_1) = P_0 = M_n$ = const, $P_1 = 0$ is realized and Eqs (3.6) and (3.8), when boundary conditions (1.34) are taken into account, respectively take the form

(3.9)

$$2\sigma_{*1}\omega_{01}\cos^2\psi_1 + Ea_1(1-2\omega_{01})e_{*1}/\cos^2\psi_1 = 12M_n/H^2, \quad 0 \le x_1 \le D \tag{3.10}$$

where expressions (3.5) and (3.7) must be taken into account in relation (3.9). (When $x_1 = 0$, the equalities correspond to a static boundary condition with respect to the moment in the case of cylindrical transverse bending (3.6) and (3.8).)

In the case of purely elastic behaviour of the binder material, Eq. (3.10), when multiplied by $\cos^2\psi_1$, reduces to a biquadratic equation in $\cos\psi_1$. Since the fibres are assumed to be entering the plate on

Table 1

Material	E, GPa	$\sigma_{0.2}$, MPa	σ_p, MPa	δ, %	ν
ADN alloy	71.0	100	150	6.0	0.31
Boron fibres	416.5	–	3150	0.2	0.23

Table 2

ω_{01}	ψ_1	$\lvert\bar{w}''\rvert$	$t_s = \lvert\bar{\varepsilon}_1^s\rvert$	t_{max}
	First (regular) solution			
0.2	12°29′	1.0490	0.1810	0.7755
0.35	40°47′	1.7443	0.1088	0.8089
	Second (singular) solution			
0.2	56°59′	3.3696	0.0563	0.2506
0.35	67°57′	7.0932	0.0268	0.2049

the edge $x_1 = 0$, then, necessarily, $\cos\psi_1 > 0$. Hence, only the two real positive roots of Eq. (3.10), which depend parametrically on ω_{01}, will be solutions of the equistressed reinforcement problem. It follows from relations (2.5) and (2.7) that $\sigma_{*1} = E_1 e_{*1}$ in the case of purely elastic bending. Dividing Eq. (3.10) by σ_{*1}, a small parameter $\lambda = E/E$, which accompanies the second term of the left-hand side of (3.10), can be separated out. By analysing the solution of the biquadratic equation (3.10) it can be shown that all the unknown functions, corresponding to the first of the solutions of this equation, when account is taken of (3.3), have finite limits when $\lambda \to 0$. (This solutions is called a "regular" solution.) The second solution of Eq. (3.10) possesses the property that $\cos^2\psi_1 \to 0$ when $\lambda \to 0$ and this means that, according to (3.3), $|w''| \to \infty$. (This solution is called a "singular" solution.)

We will investigate the solution of the problem of the equistressed reinforcement of an elongated rectangular plate with cylindrical bending made of ADN aluminium alloy reinforced with boron fibres. (The mechanical characteristics of the phase materials of the boroaluminium [12] are shown in Table 1.)

We will assume that the plate is loaded with a bending moment, the value of which is equal to $M_n = H^2\sigma_1/12(p(x_1) = F_{nz} = 0)$ ($\sigma_1^+ = \sigma_{p1}$ is the ultimate strength of the boron fibres). In this case when the behaviour of the phase materials is ideally elastic, the solution of the problem of equistressed reinforcement at all points of the structure is given by formulae (3.3) and (3.10). The values of the unknown quantities ψ_1 and $|\bar{w}''| = H|w''|/(2\varepsilon_1^+)$ ($\varepsilon_1^+ = \sigma_1^+/E_1$), obtained for $\omega_{01} = 0.2$ and $\omega_{01} = 0.35$, are presented in Table 2.

Comparison of the data presented in Table 2 shows that a change in ω_{01}, i.e. in the amount of reinforcement in the plate, leads to a significant change in the structure of the equistressed reinforcement and in the deformed state in the binder and it therefore makes senses to carry out a purposeful optimization in the set of schemes for the equistressed reinforcement of bending plates. For instance, of the four solutions presented in Table 2, the scheme corresponding to the regular solution of the problem of equistressed reinforcement when $\omega_{01} = 0.2$ will be the best from the point of view of the strength of the binder (min $|w''|$ since, in the case of purely elastic cylindrical bending, the magnitude of σ is proportional to $|w''|$, see (1.23) and (1.19)) and there is the least consumption of the reinforcement (min ω_{01}). (It generally follows from Table 2 and from numerous calculations we have carried out that equistressed reinforcement schemes which correspond to a regular solution of the problem are more effective from the point of view of the strength of the binder than schemes corresponding to the singular solution.)

It has been assumed above that the phase materials behave in an ideally elastic manner. We will now estimate the carrying capacity of real boroaluminium plates which are bent with cylindrical bending. It is obvious that, when considering purely elastic bending, the carrying capacity of a plate will be defined by the occurrence of plastic deformations in the binder and, also, by the breakdown or loss of stability of the elastically brittle boron fibres on the faces of the plate.

In order to estimate the carrying capacity of equistressed reinforced plates, we will represent the bending moment M_n in the form

$$M_n = t\overline{M}_n\sigma_1^+H^2/6, \quad \overline{M}_n = 0.5, \quad t>0 \tag{3.11}$$

where t is a loading parameter.

Using the well-known formulae [12]

$$\varepsilon_*^0 = 2\sqrt{\omega_1 E/[3(1-\omega_1)E_1]}, \quad \varepsilon_*^1 = G_m/[\omega_1[1-\omega_1]E_1]$$

We will estimate the critical deformations of the fibres at which they lose stability in a stretching mode (ε_*^0) or a shear mode (ε_*^1). Calculations showed that, in the case of the boroaluminium composition being considered over the real range of change in the reinforcement intensity ($0.05 \le \omega_1 \le 0.95$), the quantities ε_*^0 and ε_*^1 are more than twice as great as the value of the limiting strain ($\varepsilon_{p1} = \sigma_{p1}/E_1$) of the boron fibres under tension. (When calculating the quantities ε_*^0 and ε_*^1, the possibility of the occurrence of plastic deformations in the binder was taken into account.) Consequently, in the case of transverse bending, the carrying capacity of the boron fibres will be exhausted due to their fracture under tension and not due to loss of stability accompanying compression.

In order to determine the greatest value of the loading parameter (t_s) accompanying purely elastic cylindrical bending, it is necessary to estimate the limiting value of $|w_s''|$, at which the stressed state in the binder on the faces reaches the yield point σ_s. Here, $h = h_* = H$ and this means that, from expressions (3.4), we obtain

$$|w_s''| = 2\sigma_s/(HEa_1\sqrt{1-\nu+\nu^2}), \quad \sigma_s = \sigma_{0,2} \tag{3.12}$$

Since it is necessary to estimate the carrying capacity of a structure with equistressed reinforcement structures which have already been obtained, the strain modulus ε_1^s of the reinforcement on the faces of the plate, corresponding to the value $|w_s''|$ can be determined from relations (3.3) and (3.12):

$$|\varepsilon_1^s| \equiv |He_1^s/2| = |Hw_s''/2|\cos^2\psi_1 \tag{3.13}$$

Substituting expressions (3.11)–(3.13) into Eq. (3.10) and taking account of the equality $\sigma_{*1} = E_1e_1^sH$, we obtain

$$t_s = [2|\bar{\varepsilon}_1^s|\omega_{01}\cos^2\psi_1 + \lambda a_1(1-2\omega_{01})|\bar{\varepsilon}_1^s|/\cos^2\psi_1]/|\overline{M}_n| \tag{3.14}$$
$$\bar{\varepsilon}_1^s = \varepsilon_1^s/\varepsilon_1^+, \lambda = E/E_1$$

The values of t_s and $\bar{\varepsilon}_1^s$, corresponding to the equistressed reinforced structures, have been included in Table 2. The values of t_s and $|\bar{\varepsilon}_1^s|$ are identical by virtue of the meaning of these quantities (compare relations (3.10), (3.11) and (3.14)).

It follows from a comparison of the values of t_s that a plate corresponding to the regular solution of the equistressed reinforcement problem with $\omega_{01} = 0.2$ has the greatest carrying capacity and that a structure corresponding to the singular solution with $\omega_{01} = 0.35$ has the lowest carrying capacity. According to its meaning, the quantity $|\bar{\varepsilon}_1^s|$ is equal to the ratio of the stress modulus in the reinforcement $|\sigma_{01}|$ on the faces of the plate (when there are plastic deformations in the binder) to the value of the ultimate strength of the fibres σ_{p1}. It can therefore be concluded from a comparison of the magnitude of $|\bar{\varepsilon}_1^s|$ presented in Table 2 that the carrying capacity of the reinforcement in the case of the purely elastic bending of a boroaluminium plate is only slightly utilized (to the extent of less than 20%).

We will now estimate the carrying capacity of equistressed reinforced plates with cylindrical bending in which the carrying capacity of the fibres is utilized to the maximum degree, that is, we require that the stresses in the fibres on the faces of the plate should be equal in modulus to their ultimate strength ($|\sigma_{01}| = \sigma_{p1}$). Then, plastic deformations occur in the binder in the outer layers of the plate, adjacent to the faces.

In order to estimate the carrying capacity of the structures in question when the binder material behaves in an elastoplastic manner, we first need to determine the relative thickness $\bar{h} = h/H$ of the elastic layer in the binder. It follows from a comparison of expressions (3.13) and (3.7), where $|e_{*1}| = 2\varepsilon_{p1}$, that the value of $\bar{h}$ is identical with the value of $|\bar{\varepsilon}_1^s|$. (This fact is a consequence of Kirchhoff's hypothesis.) With the known value of $\bar{h}$, from Eq. (3.9), taking expressions (3.11) into account, we obtain the maximum values of the loading parameter t_{max} for the case when the binder material behaves in an elastoplastic manner

$$\begin{aligned} t_{max} &= \{2|\sigma_{*1}|H^2\omega_{01}\cos^2\psi_1 + \\ &+ (1-2\omega_{01})[(Ea_1h^3 + 4E_*(H^3-h^3)/3)|e_{*1}|/(H\cos^2\psi_1) + \\ &+ \sqrt{3}(\sigma_s - E_*\varepsilon_*)(H^2-h^2)]\}/(2|\overline{M}_n|H^2\sigma_1^+) \end{aligned} \tag{3.15}$$

where $\sigma_{*1} = 2\sigma_{p1}$, $e_{*1} = 2\varepsilon_{p1}$ (see (2.5)–(2.7) when $\sigma_{01} = \sigma_{p1} = \sigma_1^+$) and the values of ψ_1, ω_{01} and $h = \bar{h}H$ are taken from Table 2.

The values of t_{max} obtained using formula (3.15) are given in Table 2. A comparison of the values of t_s and t_{max}, which are given in the same rows of Table 2, shows that, by virtue of the greatest utilization of the carrying capacity of the fibres, the carrying capacity of the bending equistressed reinforced plates in the case of elastoplastic deformation of the binder is 4–8 times higher than the carrying capacity of the corresponding elastic structures. Here, the structure, corresponding to the regular solution with $\omega_{01} = 0.35$, possess the greatest carrying capacity in the inelastic case and not the plate corresponding to the regular solution with $\omega_{01} = 0.2$ which, in the case of purely elastic bending, is the best of all the plates considered. Consequently, equistressed reinforcement schemes which are the most effective from strength considerations in the case of purely elastic bending may not turn out to be quite so effective in the case of elastoplastic bending.

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